Contents

0 Preface **1**
Summary . . . 1
Zusammenfassung . . . 3
Acknowledgements . . . 5

1 Introduction **7**
1.1 Non-linear science . . . 7
1.1.1 Law of mass action . . . 7
1.1.2 Nonlinear systems, Bifurcations and instabilities . . . 9
1.1.3 Active elements . . . 11
1.2 Pattern formation in homogeneous systems . . . 13
1.2.1 Spatio-temporal dynamics . . . 13
1.2.2 Active media . . . 16
1.2.3 Chemical patterns . . . 17
1.2.4 Biological patterns . . . 20
1.2.5 Numerical integration of continuous models . . . 22
1.3 Pattern formation in heterogeneous systems . . . 23
1.3.1 Periodically heterogenous . . . 23
1.3.2 Randomly heterogeneous media . . . 25
1.4 Pattern formation in discrete systems . . . 25
1.4.1 Homogeneous discrete system equations . . . 26
1.4.2 Heterogeneous discrete system . . . 27

2 Generic studies **29**
2.1 Homogenization theories . . . 29
2.2 Effective medium theory . . . 31
2.2.1 Effective reactions . . . 33
2.2.2 Effective Diffusion . . . 33
2.3 Aplications to nonlinear systems, validation . . . 35
2.3.1 Bistable fronts . . . 35
2.3.2 Excitable waves . . . 36
2.4 Beyond homogenization . . . 37
2.4.1 Large size heterogeneities . . . 37

2.4.2 Time dependent heterogeneities . . . 38
2.4.3 Non-conducting heterogeneities . . . 38
2.4.4 Defects . . . 39

3 Applications 41
3.1 Chemistry . . . 41
3.1.1 Brusselator model . . . 42
3.1.2 FitzHugh–Nagumo model with two inhibitors . . . 42
3.2 Cardiology . . . 44
3.2.1 Ischemia . . . 45
3.2.2 Fibroblast . . . 46
3.2.3 Diffuse fibrosis . . . 47
3.2.4 Localized diffuse fibrosis . . . 48
3.2.5 Towards more realistic microscopic modeling . . . 50

4 Outlook and future research directions 51

Bibliography 53

A Articles on generic models of active media 61
A.1 Effective two dimensional Medium Theory . . . 63
A.2 Effective three dimensional Medium Theory . . . 69
A.3 Wave propagation in heterogeneous media . . . 81
A.4 Randomly distributed heterogeneities . . . 93

B Articles on aplications in active media 103
B.1 Wave propagation in chemical reactions in microemulsions . . . 105
B.2 Reduced discrete coupling in excitable media . . . 115
B.3 Reentry in heterogeneous discrete model . . . 125
B.4 Reentry produced by small-scale heterogeneitie . . . 137
B.5 Ectopic pacemakers in the heart . . . 155